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A RESALE EXPLANATION FOR THE DECLINING PRICE ANOMALY IN SEQUENTIAL AUCTIONS

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Abstract: This paper studies price trends in a sequential first-price common-value auction with resale. It differs from the previous research in that it considers sequential auctions with multi-unit demand. In the two-stage case, we propose a condition that guarantees the existence of a symmetric monotonic equilibrium which exhibits a declining trend. This is because bidders have the incentive to overbid in the first round to lower their rivals' intertemporal inference on the object value so that they can obtain a second-stage advantage. We also characterize the necessary properties of symmetric monotonic equilibria in the finite N-stage and the infinite-stage cases. In the former case, the price trend remains constant and drops only at the last stage; in the latter case, we have a constant price trend throughout.

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INTRODUCTION

The declining price trend in sequential auctions has long been a puzzle in auction research. Empirical findings of declining prices have been reported in such papers as Milgrom and Weber (1982b) in transponder-leases auctions; Ashenfelter (1989), McAfee and Vincent (1993) and Ginsburgh (1998) in wine auctions; Ashenfelter and Genesove (1992) in real-estate auctions; Beggs and Graddy (1997) in art auctions and van den Berg et al. (2001) in flower auctions, etc. In a sequential auction of identical objects, we normally expect a similar sale price for each object. This is analytically shown in Weber (1983). So the phenomenon that prices decline in a repeated sale of identical objects poses an anomaly, which is often termed as declining price anomaly in the auction literature.

A number of theoretical studies explain this declining price anomaly from various perspectives. From the perspective of bidder preferences, McAfee and Vincent (1993) attribute the declining price trend to the non-decreasing absolute risk aversion of bidders; and Branco (1997) to synergies. From the perspective of auction structures, Milgrom and Weber (1982b) suggest that the use of agents in auctions may explain the declining prices; Black and De Meza (1993) explain this price trend with a buyer's option, which is that the winner of the first auction has the opportunity to buy the remaining objects at the winning price; von der Fehr (1994) and

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Menezes and Monteiro (1997) relate the declining price trend to the auction participation costs. From the perspective of the nature of the objects, Engelbrecht-Wiggans (1994), Bernhardt and Scoones (1994), and Gale and Hausch (1994) explain the declining prices with heterogeneity of the objects.

A common characteristic of the above theoretical literature is that almost all of them assume bidders have single-unit demand, i.e., once a bidder obtains one unit of the object at a given stage auction, she will not participate in following stages. However, in many cases where a declining price path is detected, this assumption does not seem to be appropriate. For example, in the condominium auctions (Ashenfelter and Genesove, 1992) and flower auctions (van den Berg et al, 2001), bidders tend to be the investors, who purchase the objects not for their own consumption but for the resale values. Therefore, these bidders will participate in the auction every period as long as they remain in business. Theoretical explanations for the declining price anomaly where bidders have multi-unit demand are few in literature. The main purpose of this paper is just to fill this important theory gap.

The difficulty with the sequential auction with multi-unit demand is that bidders become asymmetric after the intertemporal inference following the first-round auction. This in general makes an analytical solution impossible. But if we dispose of the intertemporal inference by allowing each bidder to have a random draw of signal in every round, the sequential auction will then be reduced to the repeated auction. This paper proposes a sequential common-value auction framework that can both maintain certain degree of intertemporal inference and at the same time accommodate bidder symmetry.

In this paper, we obtain the following two findings. First, bidding prices decline in expectation in the two-stage sequential auction under an sufficient equilibrium condition we propose. Second, we characterize the necessary properties of symmetric monotonic equilibria in the finite N-stage and the infinite-stage cases. In the former case, the price trend remains constant and drops only at the last stage; in the latter case, we have a constant price trend throughout.[1] These results well conform to the empirical evidences. The remainder of the paper proceeds as follows. In Section 2, we first provide some empirical evidence to motivate our model. In Section 3, we set up the model. Section 4 gives the equilibrium solutions. Section 5 concludes.

EMPIRICAL EVIDENCE

Ashenfelter and Genesove (1992) and van de Berg et al. (2001) provide convincing empirical evidences for the declining price trend in sequential auctions with multi-unit demand.

Ashenfelter and Genesove (1992) analyze the data from auctions of 83 condominium units held near Princeton, New Jersey, in April 1990. They obtain both the successful bid of each condominium unit and the resale prices for some of these units. In Ashenfelter and Genesove (1992), it is clearly shown that the sequentially auctioned units display a declining trend of bids, while the resale prices more or less exhibit a constant trend.

van de Berg et al. (2001) study the sequential auctions of a particular group of roses in Aalsmeer Flower Auction located in Netherland. They investigate the auction data for the period of June 3 to August 1, 1996. In van de Berg et al. (2001), we can see that on average the bidding prices display a declining trend. Specifically, they find that the declining bid is particularly important when going from the first to the second round of a sequential auction. This justifies our approach of analyzing a two-stage auction game as the benchmark model.

The above two papers are empirical and do not offer theoretical explanations. But they strongly motivate a theoretical work for the declining bids in sequential auctions with multi-unit demand. Notice that the bidders in condominium or rose auctions are typically dealers who do not consume the products but aim for their resale values. They do not leave the auction even though they successfully win one unit. A common-value auction with multi-unit demand definitely fits these environments well. Moreover, as van de Berg et al. (2001) mention, the observations of bids in a sequential auction are typically not independent and there may be unobservable variables affecting the realized bids. This provides us clue to understanding the declining price anomaly from the angle of bidders' intertemporal inference during the auction process.

It is reasonable to assume the resale price of the object won at a given period is an aggregation of both a common fundamental and all bidders' idiosyncratic tastes.[2] Based on the evidence on the path of resale prices in Ashenfelter and Genesove (1992), the fundamental can be assumed to be a martingale and bidders' idiosyncratic tastes are assumed to be cross-period iid. This framework will make an object's current-stage resale price be parameterized by the realized resale price of the previous stage. This cross-period correlation of resale prices aims to capture the persistence of consumer preferences in the final consumption market.[3]

As we can see below, the equilibrium results derived from our model are in conformity with the above evidences. In the two-stage model, the bidding prices drop in expectation. While in the finite N-stage case, the bidding price drops in expectation only at the last stage; and in the infinite-stage case, the derived bids remain constant in expectation. Van de Berg et al (2001) find in rose auctions that at any round the bid decline is stronger if the number of remaining units is smaller. Thus, for a long sequential auction, a generally constant bids path shall be expected since the number of remaining units tends to be large.

THE MODEL

We first set up the sequential first-price common-value auction model in a two-stage environment. There is one object for sale at each period t, where t = 1, 2. There are N risk-neutral bidders. There is no reserve price. At the end of each period t, all bidders' bids submitted in this period will be announced. Bidders pursue the object not for their own consumption, but aim to resell it at the current-period resale price P_t. The value of P_t is realized immediately after the object is auctioned at period t and P_t becomes publicly observable since then. An object can always be auctioned at a given period because there is no reserve price. We assume that it can always be resold at that period's resale price P_t. Bidder 1,2,...N observe their private signals $X_t^1, X_t^2, \ldots X_t^N$ respectively at period t. All bidders' signals are iid according to the density $f(\cdot)$ over the support $[0, \omega]$. These signals represent bidders' idiosyncratic tastes for the object at a given period. We assume that P_t is the aggregation of all bidders' individual tastes and some fundamental value. The fundamental value is an unobservable random variable Θ_t, which is drawn from a density with mean $\bar{\theta}$ at period 1 and follows a martingale process at period 2, i.e., $E\Theta_2 = \theta_1$. So the fundamental value itself does not exhibit any trend in expectation. We assume $P_t = \alpha\Theta_t + (1-\alpha)U(X_t^1, X_t^2, ...X_t^N)$, where $U(\cdot)$ is both symmetric and increasing in each of its arguments. The resale price P_t is a weighted average of the fundamental and an aggregation of the collective signals represented by $U(\cdot)$.[4] We assume the fundamental value and bidders' tastes are statistically independent. We will use the lower case notations p_t, θ_t and x_t^i to denote the realized values of P_t, Θ_t and X_t^i respectively hereafter. The discount factor is δ. The generalization of the setup to arbitrary stages is straightforward—just change $E\Theta_2 = \theta_1$ to $E\Theta_{t+1} = \theta_t$ where t is any natural number smaller than the total number of stages.

EQUILIBRIUM

The Two-stage Problem

In the two-stage sequential auction, we are looking for a monotonic symmetric equilibrium.[5] The equilibrium will take the following form:

Time 1. At the first stage, bidder i observes a signal x_1^i and bids according to $\beta_1(x_1^i)$.

Time 2. The object is awarded to the bidder with the highest bid and all bids are announced. Then all bidders' private signals become common knowledge from the bid function $\beta_1(x_1^i)$.. Whenever a bid not belonging to the support $[\beta_1(0), \beta_2(\omega)]$ is observed, its associated signal will always be inferred as zero.[6]

Time 3. The unobservable fundamental value Θ_1 is realized as θ_1. The resale price p_1 is formed according to $p_1 = \alpha\theta_1 + (1-\alpha)U(x_1^1, x_1^2, ...x_1^N)$ and then becomes public information.

Time 4. The first-stage winner resells the object at the price p_1.

Time 5. At the second stage, bidder i observes a signal x_2^i and bids according to $\beta_2(p_1, x_1^1, x_1^2, ...x_1^N, x_2^i)$, where the second-stage bid function conditions on all available information.

We will start to solve for the above equilibrium from the second stage.

The Second-stage Problem

Let us assume for the moment that all bidders follow a symmetric monotonic pure-strategy bid function truthfully at the first stage. With the disclosure of all bidders' bids, the first-stage private signals $x_1^1, x_1^2, ..., x_1^N$ become common knowledge. Since p_1 is publicly observable at the second stage, the unobservable θ_1 can be inferred from the equation of resale price formation described above. Bidders' signals are cross-period independent, while Θ_1 is correlated with Θ_2, so the only valuable information bidders will condition on at the second stage is the inferred true value of θ_1. The first-stage information $p_1, x_1^1, x_1^2, ... x_1^N$ will affect the bid only through the aggregated variable θ_1. Then the second-stage problem becomes a standard one-shot auction with public information. Now we introduce some new notation: $X_t^{-i} = \{X_t^1, X_t^2, ... X_t^N\} \setminus \{X_t^i\}$; $Y_t^i = \max_{s \neq i} X_t^s$. Then the following proposition is immediate.

Proposition 1: Assuming that a symmetric monotonic equilibrium exists, the second-stage equilibrium bid function

$$\beta_2\left(p_1, x_1^i, x_1^{-i}, x_2^i\right) = p_1 - (1-\alpha)U(x_1^i, x_1^{-i}) + (1-\alpha)E_{Y_2^i}\{E_{X_2^{-i}}[U(Y_2^i, X_2^{-i}) \mid Y_2^i] \mid Y_2^i < x_2^i\}$$

Proof. See the Appendix.

It is easy to see that the above second-stage bid function is monotonically increasing in a bidder's second-stage signal but decreasing in all bidders' first-stage signals. In our model, this feature is the key that leads to a declining price path as we will see soon.

The First-stage Problem

Now we start to solve for the first stage equilibrium bid function. According to the second-stage bid function in Proposition 1, we observe that if bidder i mimics a higher type than her true one in the first stage, her opponents will bid lower in the second stage. This fact can be understood through bidder i's opponents' intertemporal inference of the unobservable fundamental. Once the opponents observe a high bid at the first stage, i.e., a high signal from bidder i, their inference of the first-stage fundamental value will be low for a given resale price p_1 according to its functional form. Since the fundamental is a martingale, the opponents will be induced to believe that the mean of the second-stage fundamental is low hence bid low. So bidder i's logic for mimicking a higher type at the first stage can be described as follows:

> "I know that I may bid a little bit higher than the expected resale value of the object at the first stage. But I just want to create a false image of narrow margin between my bid and the realized resale price. Then my opponents will believe that this object's resale value does not quite live up to my high taste, hence may have a low fundamental. They then bid low at the second stage under their pessimistic expectations and I can easily win the object with a low bid at that time. So my loss from overbidding in the first stage can be compensated by the gain from underbidding in the second stage."

In equilibrium of course, each bidder will have to bid truthfully. The above argument on bidders' intertemporal incentives explains why in equilibrium the truthful bidding in the first stage can turn out to be more aggressive than that in the second stage.

The formal equilibrium derivation is outlined as follows. At the first stage, bidder i with a private signal x_1^i will maximize the expected overall two-stage payoff $\Pi = \Pi_1 + \delta E_{X_1^2,...,X_1^N,\Theta_1,X_2^1}\Pi_2$, where Π_1 and Π_2 are the first and second-stage payoffs respectively. We assume bidder i mimics a type $z \neq x_1^i$ at the first stage. Then following any pair of (z, x_1^i), there is an expected continuation optimal second-stage payoff $E_{X_1^2,...,X_1^N,\Theta_1,X_2^1}\Pi_2(z, x_1^i)$ for bidder i. So the first-stage bid function will be derived in such a way that setting $z = x_1^i$ will maximize the overall payoff Π. Let $[E_{X_1^{-i}}U(.,.)]'$ represent the derivative with respect to the first argument of the function U(.,.). Then the following proposition gives the first-stage equilibrium bid function.

Proposition 2: Assuming that a symmetric monotonic equilibrium exists, the first-stage equilibrium bid function

$$\beta_1\left(x_1^i\right) = \alpha\bar{\theta} + (1-\alpha)E_{Y_1^i}\{E_{X_1^{-i}}[U(Y_1^i, X_1^{-i}) \mid Y_1^i] + \frac{\delta\, E_{X_1^{-i}}U(Y_1^i, X_1^{-i})]'}{N[F^{N-1}\left(Y_1^i\right)]'} \mid Y_1^i < x_1^i\}.$$

Proof. See the Appendix.

We have derived both stages' bid functions by assuming a symmetric monotonic equilibrium exists, the key point that remains to be checked is whether this pair of bid functions indeed constitute a symmetric monotonic equilibrium. The following proposition gives the answer.

A Sufficient Equilibrium Condition

This section shows that a symmetric monotonic equilibrium exists for certain types of distribution F(·) and aggregation function U(·). The following proposition states a sufficient equilibrium condition and confirms the declining equilibrium price trend.

Proposition 3: When the expression $\frac{[E_{X_1^{-i}}U(X_1^i, X_1^{-i})]'}{[F^{N-1}\left(X_1^i\right)]'}$ is increasing in X_1^i, the bid functions derived in the above two propositions constitute a symmetric monotonic equilibrium. Also, $E_{X_1^i}\beta_1\left(X_1^i\right) > E_{P_1, X_1^i, X_1^{-i}, X_2^i}\beta_2\left(P_1, X_1^i, X_1^{-i}, X_2^i\right)$.

Proof. See the Appendix.

Mathematically, the monotonicity of $\frac{[E_{X_1^{-i}}U(X_1^i, X_1^{-i})]'}{[F^{N-1}\left(X_1^i\right)]'}$ guarantees the monotonicity of the first-stage bid function, which in turn ensures the monotonicity of the second-stage bid function. Under the above equilibrium condition, the bidding prices will exhibit declining trend. Intuitively, this sufficient equilibrium condition can be understood through the analysis of bidders' incentives. On the one hand, as we argued in Section 4.1.2., bidders have the incentive to "overbid" in the first stage in order to obtain the second-stage advantage. The term $[E_{X_1^{-i}}U(X_1^i, X_1^{-i})]'$ is just reduced from the expression that measures the gain from the first-stage over-bidding. On the other hand, it is commonly known that bidders should shade their bids sufficiently below their signals in a common-value auction to avoid the winner's curse. The term $[F^{N-1}\left(X_1^i\right)]'$ represents the winning probability and can be considered as a measure for the loss from the first-stage over-bidding due to an exacerbated winner's curse. The first-stage bidders then face these two conflicting incentives and they need to evaluate their aggregated effect, which is measured by the quotient of these two terms. To preserve the pure-strategy solution to the monotonic decision problem at the first stage, it is natural to require that this overall effect to be monotonic in each bidder's type. At the second stage, there is no such conflicting incentives, hence the equilibrium solution is standard.

It is easy to check that the difference between $E_{X_1^i}\beta_1\left(X_1^i\right)$ and $E_{P_1,X_1^i,X_1^{-i},X_2^i}\beta_2\left(P_1,X_1^i,X_1^{-i},X_2^i\right)$ is $\frac{\delta(1-\alpha)}{N}E_{X_1^i,Y_1^i}\left\{\frac{[E_{X_1^{-i}}U(Y_1^i,X_1^{-i})]'}{[F^{N-1}(Y_1^i)]'}|Y_1^i<X_1^i\right\}$. From the expression of this difference, we can see that the price drop tends to be more severe when we have a larger δ or a smaller α. Bigger δ means that gaining the second-stage advantage is more important for the overall payoff. In order to obtain bigger second-stage advantage, bidders should bid more aggressively in the first stage, leading to a larger price decline. Smaller α implies that the individual tastes are more important in the formation of the resale price. So the strategic revelation of bidders' types can affect the total payoff in a higher degree, which in turn generates a larger price drop. The above result of declining prices will still remain valid in a sequential second-price common-value auction and other auction variants as long as the incentive structure described in Section 4.1.2 is preserved.

GENERALIZATION

We then ask whether the above framework can be generalized to finite N(N > 2) or infinite stages and whether the declining price trend is still preserved there. In principle, our setup can accommodate an analytical solution for the finite-stage problem since bidders remain symmetric at any given round after processing the information from all previous rounds. Then a backward induction will derive each stage's bid function. However, this backward induction is tractable only when we know the specific forms of the aggregation function U(·) and the densit f(·). In a two-bidder three-stage example, where f(·) is a uniform density over [0,1] and $P_t=\frac{1}{3}(X_t^1+X_t^2)+\frac{2}{3}\Theta_t$, we can find that the equilibrium bid functions are $\beta_1\left(x_1^1\right)=\frac{1}{3}x_1^1+\frac{2}{3}\overline{\theta}+\frac{\delta}{6}$, $\beta_2\left(x_2^1,\theta_1\right)=\frac{1}{3}x_2^1+\frac{2}{3}\theta_1+\frac{\delta}{6}$ and $\beta_3\left(x_3^1,\theta_2\right)=\frac{1}{3}x_3^1+\frac{2}{3}\theta_2$. Replacing $\frac{2}{3}\theta_t$ with $p_t-\frac{1}{3}(x_t^1+x_t^2)$ for t = 1, 2, we can obtain the final expressions for the three bid functions.

When U(·) and f(·) remain general, the backward induction becomes intractable as the number of stages is more than 2. It is not clear whether a symmetric monotonic equilibrium exists for the more-than-two-stage problem. But we can still obtain some equilibrium properties if an equilibrium exits, which are summarized in the following proposition.

Proposition 4: When a symmetric monotonic equilibrium exists in the finite N-stage ($N > 2$) sequential auction, the equilibrium has the property: $E\beta_1(.) = E\beta_2(.)... = E\beta_{N-1}(.) > E\beta_N(.)$; when a stationary symmetric monotonic equilibrium exists in the infinite-stage sequential auction, the equilibrium has the property: $E\beta_t(.) = E\beta_{t+1}(.)\,\forall t \in \{1, 2, ...\infty\}$.

Proof: See the Appendix.

The above proposition shows that for a finite-stage problem, the bidding price drops in expectation only at the last stage. The reason for this phenomenon hinges on the Markovian property of the fundamental and the cross-period independence of bidders' signals. These two factors make a bidder able to affect her rivals' intertemporal inference only one period ahead. Then in equilibrium, the gain from affecting future payoff will be the same for all periods (since only the next period matters) except the last one (since there is no next period), hence generating a constant price trend with a price drop only at the last stage. In the infinite-stage case, there is no last stage throughout, so the price trend remains constant.

The intuition derived from the analysis of Proposition 4 gives rise to the following conjecture, which will be left for future study. We conjecture that the length of the cross-period persistence of the object fundamental plays an important role in determining the declining price trend. In a two-stage sequential auction, the fundamental is persistent across both stages and the intertemporal inference generates a definite declining price trend. Now let us consider a more general model where $P_t = U\left(\Theta_t.\theta_{t-1},...\theta_{t-s}, X_t^i, X_t^{-i}\right)$, i.e., the resale price not only depends on the current period fundamental but also the fundamentals in previous periods. Then if the fundamental is more persistent than a martingale, the current fundamental will be affected by the realized fundamentals from more-than-one previous rounds. This general model seems to be hard to solve analytically at the moment. But its solution may be conjectured from the learning interpretation obtained from the martingale case.

For example, if the fundamental is persistent across all stages, then at stage t, bidders can always affect more future stages hence bigger future payoff than at stage t + 1. Therefore, bidders will have higher incentives to raise their bids at stage t than stage t + 1, which may lead to a continuously declining price path. In another example, if the fundamental is persistent only for 3 stages while the sequential auction has 5 rounds, then we would expect $E\beta_1(.) = E\beta_2(.) = E\beta_3(.) > E\beta_4(.) > E\beta_5(.)$. This is because bidders can affect future 2 rounds' payoff (high future payoff) at stage 1, 2 and 3, while at stage 4 they can only affect 1 future round (medium future payoff) and at stage 5 zero round (low future payoff), then bidders' intertemporal decisions similar as those discussed in Section 4.1.2 tend to yield the stated price trend. In a third example, if the auction rounds are far more than the length of the persistence of the fundamental, like 50 rounds with a martingale fundamental, we should then expect an relatively constant price trend. This conforms to the findings in rose auctions in van den Berg et al (2001). Therefore, price trends with varying degrees of declining can all be generated from adjusting the length of the persistence of the fundamental. The implication is that to predict the bidding prices path in sequential auction with multi-unit demand, we shall delve into how persistent the object fundamental is.

CONCLUSION

This paper studies price trends in a sequential first-price common-value auction with resale. It differs from the previous research in that it considers sequential auctions with multi-unit demand. In the two-stage case, we propose a condition that guarantees the existence of a symmetric monotonic equilibrium which exhibits a declining trend. This is because bidders have the incentive to overbid in the first round to lower their rivals' intertemporal inference on the object value so that they can obtain a second-stage advantage. We also characterize the necessary properties of symmetric monotonic equilibria in the finite N-stage and the infinite-stage cases. In the former case, the price trend remains constant and drops only at the last stage; in the latter case, we have a constant price trend throughout. These results well conform to the empirical evidences. Future work will be devoted to showing the existence of equilibrium in general N-stage (N > 2) sequential auction and to investigating price trends when the fundamental is more persistent than a martingale.

Appendix: Proofs

Proof of Proposition 1: Let bidder 1 be the generic bidder for our derivation of the equilibrium bid functions. Since bidders' signals are cross-period independent, while Θ_1 is correlated with Θ_2, the only valuable information the bidders will condition on is the inferred true value of θ_1. Then the second-stage bid function will take the form of $\beta_2\left(x_2^1,\theta_1\right)$, where the first-stage information $p_1, x_1^1, x_1^2, \ldots x_1^N$ affects the bid only through the aggregated variable θ_1.

Let $v_2^1\left(x_2^1, y_2^1, \theta_1\right) = E[P_2 \mid X_2^1 = x_2^1, Y_2^1 = y_2^1, \Theta_1 = \theta_1]$, where $Y_2^1 = \max_{s \neq 1} X_2^s$ and y_2^1 is its realized value. The c.d.f. of Y_2^1 is $F^{N-1}(\cdot)$ and the p.d.f of Y_2^1 is $(N-1)\, F^{N-2}(\cdot)\, f(\cdot)$. Then

$$
\begin{aligned}
& E[P_2 \mid X_2^1 = x_2^1, Y_2^1 = y_2^1, \Theta_1 = \theta_1] \\
& = E[\alpha\Theta_2 + (1-\alpha)U(X_2^1, X_2^2, \ldots X_2^N) \mid X_2^1 = x_2^1, Y_2^1 = y_2^1, \Theta_1 = \theta_1] \\
& = \alpha\theta_1 + (1-\alpha)E[U(x_2^1, X_t^2, \ldots X_t^N) \mid Y_2^i = y_2^1]
\end{aligned}
$$

Let us assume bidder 1 mimics type z at the second stage given her true signal is x_2^1. Then her second-stage payoff is:

$$\Pi_2\left(z, x_2^1, \theta_1\right) = \int_0^z [v_2^1\left(x_2^1, y_2^1, \theta_1\right) - \beta_2\left(z, \theta_1\right)](N-1)F^{N-2}\left(y_2^1\right) f\left(y_2^1\right) dy_2^1$$

The first-order condition w.r.t. z leads to:

$$[v_2^1\left(x_2^1, z, \theta_1\right) - \beta_2\left(z, \theta_1\right)](N-1)F^{N-2}(z)\, f(z) - \beta_2'\left(z, \theta_1\right)F^{N-1}(z) = 0 \tag{1}$$

The equilibrium condition $z = x_2^1$ leads to:

$$[v_2^1\left(x_2^1, x_2^1, \theta_1\right) - \beta_2\left(x_2^1, \theta_1\right)](N-1)\, f\left(x_2^1\right) - \beta_2'\left(x_2^1, \theta_1\right)F\left(x_2^1\right) = 0. \tag{2}$$

The solution to equation (2) is standard, which gives:

$$\begin{aligned}&\beta_2\left(x_2^1,\theta_1\right)\\&=\frac{1}{F^{N-1}\left(x_2^1\right)}\int_0^{x_2^1} v_2^1\left(y,y,\theta_1\right)dF^{N-1}\left(y\right)\\&=E\left(v_2^1\left(Y_2^1,Y_2^1,\theta_1\right)\mid Y_2^1<x_2^1\right)\\&=\alpha\theta_1+\left(1-\alpha\right)E\{E[U(Y_2^1,X_t^2,...X_t^N)\mid Y_2^1]\mid Y_2^1<x_2^1\}\end{aligned}$$

We can obtain the final equilibrium bid function by replacing θ_1 with $\frac{p_1}{\alpha}-\frac{(1-\alpha)U(x_1^1,x_1^2,...x_1^N)}{\alpha}$. Q.E.D.

Proof of Proposition 2: Let us assume all bidders follow the monotonic bid functions $\beta_1(\cdot)$ and $\beta_2(\cdot)$ truthfully except bidder 1. At the signal x_1^1, let us assume bidder 1 mimics $z\neq x_1^1$ at the first stage. Let $v_1^1\left(x_1^1,y_1^1\right)=E\left[P_1\mid X_1^1=x_1^1,Y_1^1=y_1^1\right]$, where $Y_1^1=\max_{s\neq 1} X_1^s$ and y_1^1 is its realized value. Then her first-stage payoff is: $\Pi_1\left(x_1^1,z\right)=\int_0^z\left(v_1^1\left(x_1^1,y_1^1\right)-\beta_1\left(z\right)\right)\left(N-1\right)F^{N-2}\left(y_2^1\right)f\left(y_2^1\right)dy_1^1$.

At the second stage, all other bidders will follow $\beta_2\left(x_2^i,\hat{\theta}_1\right)$, where $i\neq 1$. Notice that the true $\theta_1=\frac{p_1}{\alpha}-\left(\frac{1}{\alpha}-1\right)U\left(x_1^1,x_1^2,...x_1^N\right)$, where θ_1 is increasing in p_1 and decreasing in all other arguments. But bidder 1 mimics type z, so all other bidders will be induced to believe that the realized fundamental is $\hat{\theta}_1=\frac{p_1}{\alpha}-\left(\frac{1}{\alpha}-1\right)U\left(z,x_1^2,...x_1^N\right)\neq\theta_1$. Therefore, bidder 1's second-stage payoff is:

$$\Pi_2\left(x_2^1,\tau,\theta_1,\hat{\theta}_1\right)=\int_0^{\tau}[v_2^1\left(x_2^1,y_2^1,\theta_1\right)-\beta_2(\tau,\hat{\theta}_1)]\left(N-1\right)F^{N-2}\left(y_2^1\right)f\left(y_2^1\right)dy_2^1.$$

Notice that we let bidder 1 bid according to $\beta_2\left(.,\hat{\theta}_1\right)$ rather than $\beta_2(.,\theta_1)$ even though she knows the true value θ_1. This is because it is never optimal for bidder 1 to bid outside the support of $\beta_2\left(.,\hat{\theta}_1\right)$, according to which all her rivals bid. Then Bidder 1's decision is simply to choose a bid within this particular support, which corresponds to a certain type τ in [0, ω]. So $\tau^*=\arg\max_{\tau}\Pi_2\left(x_2^1,\tau,\theta_1,\hat{\theta}_1\right)$. The FOC w.r.t. τ leads to:

$$[v_2^1\left(x_2^1,\tau,\theta_1\right)-\beta_2\left(\tau,\hat{\theta}_1\right)]\left(N-1\right)f\left(\tau\right)-\beta_2'\left(\tau,\hat{\theta}_1\right)F\left(\tau\right)=0 \quad (3)$$

Let $v_2^1\left(y,y,\theta_1\right)\left(N-1\right)F^{N-2}\left(y\right)f\left(y\right)=V\left(y,\theta_1\right)$.

Then $\beta_2\left(x_2^1,\theta_1\right)=\frac{1}{F^{N-1}\left(x_2^1\right)}\int_0^{x_2^1}V\left(y,\theta_1\right)dy$.

So $$\beta_2'\left(x_2^1,\theta_1\right)=\frac{V\left(x_2^1,\theta_1\right)}{F^{N-1}\left(x_2^1\right)}-\frac{(N-1)\,f\left(x_2^1\right)\beta_2\left(x_2^1,\theta_1\right)}{F\left(x_2^1\right)}.$$

Substituting the functional form of $\beta_2'\left(x_2^1,\theta_1\right)$ into the above equation (3), we have:

$$v_2^1\left(x_2^1,\tau,\theta_1\right)=v_2^1\left(\tau,\tau,\hat{\theta}_1\right) \tag{4}$$

Equation (4) can be transformed into:

$$\begin{aligned}&\alpha\theta_1+\frac{1-\alpha}{F^{N-1}(\tau)}\int_0^\tau\cdots\int_0^\tau U\left(x_2^1,x_2^2\ldots x_2^N\right)f\left(x_2^2\right)\ldots f\left(x_2^N\right)dx_2^2\ldots dx_2^N\\&=\alpha\hat{\theta}_1+\frac{1-\alpha}{F^{N-1}(\tau)}\int_0^\tau\cdots\int_0^\tau U\left(\tau,x_2^2\ldots x_2^N\right)f\left(x_2^2\right)\ldots f\left(x_2^N\right)dx_2^2\ldots dx_2^N\end{aligned} \tag{5}$$

Equation (5) can be reduced to:

$$\begin{aligned}&-U(x_1^1,x_1^2,\ldots x_1^N)+\frac{1}{F^{N-1}(\tau)}\int_0^\tau\cdots\int_0^\tau U\left(x_2^1,x_2^2\ldots x_2^N\right)f\left(x_2^2\right)\ldots f\left(x_2^N\right)dx_2^2\ldots dx_2^N\\&=-U(z,x_1^2,\ldots x_1^N)+\frac{1}{F^{N-1}(\tau)}\int_0^\tau\cdots\int_0^\tau U\left(\tau,x_2^2\ldots x_2^N\right)f\left(x_2^2\right)\ldots f\left(x_2^N\right)dx_2^2\ldots dx_2^N\end{aligned} \tag{6}$$

Given that $U(\cdot)$ is both monotonic and symmetric in each of its components, we can obtain that for any z together with the set of realized signals $\left\{x_2^1,x_1^1,x_1^2,\ldots x_1^N\right\}$, there is a unique optimal $\tau^*\left(x_2^1,z,x_1^1,x_1^2,\ldots x_1^N\right)$ that corresponds to z. Also, $\tau^*\left(x_2^1,z,x_1^1,x_1^2,\ldots x_1^N\right)$ is monotonic in x_1^1. When $z\to x_1^1,\tau^*\to x_2^1$, so τ^* will not be a corner solution over the support [0, ω] with probability 1.

The overall two-stage payoff is: $\Pi=\Pi_1+\delta E_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\Pi_2$. Π_1 is bidder 1's first-stage payoff given she mimics type z. Π_2 is her optimal second-stage payoff by mimicking type $\tau^*\left(x_2^1,z,x_1^1,x_1^2,\ldots x_1^N\right)$ after processing all the interperiod information. Then the FOC. w.r.t. z is:

$$\frac{\partial\Pi_1}{\partial z}+\delta\frac{dE_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\Pi_2^*}{dz}=0 \tag{7}$$

It is easy to see that $\frac{\partial\Pi_1}{\partial z}=[v_1^1\left(x_1^1,z\right)-\beta_1(z)](N-1)F^{N-2}(z)\,f(z)-\beta_1'(z)F^{N-1}(z)$ and

$\frac{dE_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\Pi_2}{dz}=E_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\left(\frac{d\Pi_2}{dz}\right)$. Using the envelope theorem without constraint (since we argue before that the optimal τ^* will not be a corner solution with probability 1), we have:

$$\begin{aligned}\frac{d\Pi_2}{dz}&=\int_0^{\tau^*}-\frac{\partial\beta_2(\tau^*,\hat{\theta}_1)}{\partial\hat{\theta}_1}\frac{\partial\hat{\theta}_1}{\partial z}(N-1)F^{N-2}\left(y_2^1\right)f\left(y_2^1\right)dy_2^1\\&=(1-\alpha)\frac{\partial U(z,x_1^2,\ldots x_1^N)}{\partial z}F^{N-1}\left(\tau^*\left(x_2^1,z,x_1^1,x_1^2,\ldots x_1^N\right)\right)\end{aligned}$$

So $\frac{d\Pi_2}{dz} > 0$ and $E_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\left(\frac{d\Pi_2}{dz}\right) > 0$. Let $E_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\left(\frac{d\Pi_2}{dz}\right) = H\left(z, x_1^1\right)$. Then we have

$$H\left(x_1^1, x_1^1\right) = E_{x_1^2,\ldots,x_1^N,\Theta_1,x_2^1}\left(\frac{d\Pi_2}{dz}\right)|_{z=x_1^1}$$
$$= (1-\alpha)E_{x_2^1}\left\{E_{x_1^2,\ldots,x_1^N}\left[\frac{\partial U(z, x_1^2, \ldots x_1^N)}{\partial z}F^{N-1}\left(\tau^*\left(x_2^1, z, x_1^1, x_1^2, \ldots x_1^N\right)\right)\right]\right\}|_{z=x_1^1}$$

From the above derivation, we know that $\tau^*\left(x_2^1, z, x_1^1, x_1^2, \ldots x_1^N\right)|_{z=x_1^1} = x_2^1$, $H\left(x_1^1, x_1^1\right)$ can thus be further simplified as:

$$H\left(x_1^1, x_1^1\right)$$
$$= (1-\alpha)E_{x_2^1}\left\{E_{x_1^2,\ldots,x_1^N}\left[\frac{\partial U(z, x_1^2, \ldots x_1^N)}{\partial z}F^{N-1}\left(x_2^1\right)\right]\right\}$$
$$= (1-\alpha)E_{x_1^2,\ldots,x_1^N}\left[\frac{\partial U(z, x_1^2, \ldots x_1^N)}{\partial z}E_{x_2^1}F^{N-1}\left(x_2^1\right)\right]$$
$$= \frac{1-\alpha}{N}E_{x_1^2,\ldots,x_1^N}\left[\frac{\partial U(x_1^1, x_1^2, \ldots x_1^N)}{\partial z}\right]$$
$$= \frac{1-\alpha}{N}\left[\frac{\partial E_{x_1^2,\ldots,x_1^N}[U(x_1^1, x_1^2, \ldots x_1^N)]}{\partial x_1^1}\right]$$

In equilibrium, we need $z = x_1^1$, so the FOC of equation (7) becomes:

$$[v_1^1\left(x_1^1, x_1^1\right) - \beta_1\left(x_1^1\right)](N-1)F^{N-2}\left(x_1^1\right)f\left(x_1^1\right) - \beta_1'\left(x_1^1\right)F^{N-1}\left(x_1^1\right) + \delta H\left(x_1^1, x_1^1\right) = 0 \quad (8)$$

Equation (8) can be rearranged as

$$[(v_1^1\left(x_1^1, x_1^1\right) + \frac{\delta H\left(x_1^1, x_1^1\right)}{(N-1)F^{N-2}\left(x_1^1\right)f\left(x_1^1\right)}) - \beta_1\left(x_1^1\right)](N-1)f\left(x_1^1\right) - \beta_1'\left(x_1^1\right)F\left(x_1^1\right) = 0 \quad (9)$$

Let $\hat{V}\left(y_1^1, y_1^1\right) = \left[v_1^1\left(y_1^1, y_1^1\right) + \frac{\delta H\left(y_1^1, y_1^1\right)}{(N-1)F^{N-2}\left(y_1^1\right)f\left(y_1^1\right)}\right]$. *Then* $\beta_1\left(x_1^1\right) = E[\hat{V}\left(Y_1^1, Y_1^1\right) | Y_1^1 < x_1^1]$

Replacing $v_1^1\left(y_1^1, y_1^1\right)$ with $\alpha E\Theta_1 + (1-\alpha)E[U(Y_1^1, X_1^{-1}) | Y_1^1 = y_1^1]$, we can obtain the desired expression. Q.E.D.

Proof of Proposition 3: The monotonicity of $\frac{\left[E_{X_1^{-i}}U(X_1^i, X_1^{-i})\right]'}{\left[F^{N-1}\left(X_1^i\right)\right]'}$ ensures the monotonicity of the derived first-stage bid function. Now we will check whether the bid functions derived in Proposition 1 and 2 indeed constitute an equilibrium. We need to show that given all other bidders follow these monotonic bid functions, bidder 1 has no incentive to mimic a false type. Let us assume bidder 1 observes a type y while decides to mimic type z in the first stage and let her total expected payoff (the first-stage payoff plus the expected second-stage optimal continuation payoff) be Π. Then $\frac{d\Pi}{dz} = [v_1^1(y,z) - \beta_1(z)](N-1)F^{N-2}(z)\,f(z) - \beta_1'(z)F^{N-1}(z) + \delta H(z,y)$ according to equation (7) in the proof of Proposition 2. The right-hand side of the above equation can be rearranged as $(N-1)F^{N-2}(z)\,f(z)[v_1^1(y,z) + \frac{\delta H(z,y)}{(N-1)F^{N-2}(z)\,f(z)} - \beta_1(z)] - \beta_1'(z)F^{N-1}(z)$. We next need to show that the term $v_1^1(y,z) + \frac{\delta H(z,y)}{(N-1)F^{N-2}(z)\,f(z)}$ is monotonic in y.

Since both $v_1^1(y,z)$ and H(z, y) are monotonic in y as we can see from the proof of Proposition 2, the whole term is monotonic in y too. Then using equation (8) in the proof of Proposition 2, it is easy to show that setting $z < y$ makes $\frac{d\Pi}{dz} > 0$ and $z > y$ makes $\frac{d\Pi}{dz} < 0$. So Π is maximized by choosing z = y. The off-equilibrium path belief also eliminates a bidder's incentive to bid above $\beta_1(\omega)$ in the first stage because she will be inferred as zero type, which always makes her rivals bid more aggressively at the second stage, hence lowering the bidder's overall payoff. Given that bidders bid truthfully in the first stage, the second-stage problem is a standard common-value auction with public information, whose equilibrium check is standard, hence omitted.

A comparison of each stage's bid function shows that for the same signal, the first-stage bid function is bigger than the second-stage one by the term $\frac{\delta(1-\alpha)}{N} E_{Y_1^i}\left\{\frac{\delta[E_{X_1^{-i}} U(Y_1^i, X_1^{-i})]'}{[F^{N-1}(Y_1^i)]'} \mid Y_1^i < X_1^i\right\}$. Q.E.D.

Proof of Proposition 4: We will first consider the finite N-stage case. Again, we let bidder 1 be our generic bidder and assume a symmetric monotonic equilibrium exists. Then in equilibrium, i.e., given truthful bidding, we must have $E\beta_{N-1}(\cdot) > E\beta_N(\cdot)$ (N is the last stage) from our proof for the two-stage problem. Now at period t + 1, where $t \in \{1, 2, \ldots N-1\}$, if bidder 1 observes a signal x_{t+1}^1 and mimics a type z, given all other bidders follow the equilibrium strategy, she can obtain a best continuation payoff $c(x_{t+1}^1, z, p_t, x_t^{-1})$. Let $v_t^1(x_t^1, y_t^1, \theta_{t-1}) = E\left[P_t \mid X_t^1 = x_t^1, Y_t^1 = y_t^1, \Theta_{t-1} = \theta_{t-1}\right]$, where $Y_t^1 = \max_{s \neq 1} X_t^s$ and y_t^1 is its realized value. Then bidder 1's stage payoff of period t + 1 is:

$$\Pi_{t+1}(z, x_{t+1}^1, \theta_t) = \int_0^z \left[v_{t+1}^1(x_{t+1}^1, y_{t+1}^1, \theta_t) - \beta_{t+1}(z, \theta_t)\right](N-1)F^{N-2}(y_{t+1}^1)\,f(y_{t+1}^1)\,dy_{t+1}^1$$

At period t, if bidder 1 observes a signal x_t^1 and mimics a type γ, given all other bidders follow the equilibrium strategy, she will mimic a type z at period t + 1 given signal x_{t+1}^1 and obtain a best continuation payoff $c(x_{t+1}^1, z, p_t, x_t^{-1})$.[7] Bidder 1's stage payoff of period t is: $\Pi_t(\gamma, x_t^1, \theta_{t-1}) = \int_0^\gamma \left[v_t^1(x_t^1, y_t^1, \theta_{t-1}) - \beta_t(\gamma, \theta_{t-1})\right](N-1)F^{N-2}(y_t^1)\,f(y_t^1)\,dy_t^1$. Given bidder 1 mimics γ at period t, her rivals' inference of $\hat{\theta}_t$ will be $\frac{p_t}{\alpha} - \left(\frac{1}{\alpha} - 1\right)U(\gamma, x_t^{-1})$ and her stage payoff of period t + 1 will become:

$$\Pi_{t+1}\left(z, x_{t+1}^1, \theta_t, \hat{\theta}_t\right) = \int_0^z [v_{t+1}^1\left(x_{t+1}^1, y_{t+1}^1, \theta_t\right) - \beta_{t+1}\left(z, \hat{\theta}_t\right)](N-1)F^{N-2}\left(y_{t+1}^1\right) f\left(y_{t+1}^1\right) dy_{t+1}^1$$

Then bidder 1's total payoff starting from period t on is:

$$\int_0^\gamma [v_t^1\left(x_t^1, y_t^1, \theta_{t-1}\right) - \beta_t\left(\gamma, \theta_{t-1}\right)](N-1)F^{N-2}\left(y_t^1\right) f\left(y_t^1\right) dy_t^1$$
$$+\delta E_{,X_{t+1}^1, X_t^{-1}, P_t} \int_0^z [v_{t+1}^1\left(x_{t+1}^1, y_{t+1}^1, \theta_t\right) - \beta_{t+1}\left(z, \hat{\theta}_t\right)](N-1)F^{N-2}\left(y_{t+1}^1\right) f\left(y_{t+1}^1\right) dy_{t+1}^1$$
$$+\delta^2 E_{X_{t+1}^1, X_t^{-1}, P_t} c(x_{t+1}^1, z, p_t, x_t^{-1})$$

\\ Then the FOC w.r.t. γ leads to:

$$\left[v_t^1\left(x_t^1, \gamma, \theta_{t-1}\right) - \beta_t\left(\gamma, \theta_{t-1}\right)\right](N-1)F^{N-2}(\gamma) f(\gamma) - \beta_t'\left(\gamma, \theta_{t-1}\right)F^{N-1}(\gamma)$$
$$-\delta E_{X_{t+1}^1, X_t^{-1}, P_t}\left[\frac{\partial \beta_{t+1}(z, \hat{\theta}_t)}{\partial \hat{\theta}_t} \frac{\partial \hat{\theta}_t}{\partial \gamma}(N-1)F^{N-2}\left(y_{t+1}^1\right) f\left(y_{t+1}^1\right) dy_{t+1}^1\right]\Big|_{z^*} = 0 \qquad (10)$$

Equation (10) is a necessary condition for any equilibrium. Setting $\gamma = x_t^1$, then the optimal z^* must equal x_{t+1}^1 as in the proof of Proposition 2. So equation (10) can be transformed to:

$$\left[v_t^1\left(x_t^1, x_t^1, \theta_{t-1}\right) - \beta_t\left(x_t^1, \theta_{t-1}\right)\right](N-1)F^{N-2}\left(x_t^1\right) f\left(x_t^1\right) - \beta_t'\left(x_t^1, \theta_{t-1}\right)F^{N-1}\left(x_t^1\right)$$
$$+\delta E_{X_{t+1}^1, X_t^{-1}, \Theta_t}\left[\frac{\partial \beta_{t+1}(z, \hat{\theta}_t)}{\partial \hat{\theta}_t}\left(\frac{1}{\alpha} - 1\right)\frac{\partial U(x_t^1, x_t^{-1})}{\partial x_t^1} F^{N-1}\left(x_{t+1}^1\right)\right] = 0 \qquad (11)$$

In the finite N-stage case, $\beta_N(x_N^1, \theta_{N-1})$ can be obtained in the same way as in the proof of Proposition 1, where θ_{N-1} is linearly separable with coefficient α from all other parts of the expression of the bid function. So $\frac{\partial \beta_N\left(x_N^1, \hat{\theta}_{N-1}\right)}{\partial \hat{\theta}_{N-1}}$ equals the constant α. Similarly, from equation (11) we can show that $\frac{\partial \beta_t\left(x_t^1, \hat{\theta}_{t-1}\right)}{\partial \hat{\theta}_{t-1}} = \alpha$ for all $t < N$ with backward induction. This leads to the conclusion that $E\beta_t(\cdot)$ will be the same for all period t except the last one since the last stage does not have a continuation payoff anymore.

We then consider the infinite-stage case. We assume there exists a stationary symmetric monotonic equilibrium, and then the payoff from period t on under valuation x_t^1 with mimicking type γ equals: $\int_0^\gamma [v_t^1\left(x_t^1, y_t^1, \theta_{t-1}\right) - \beta_t\left(\gamma, \theta_{t-1}\right)](N-1)F^{N-2}\left(y_t^1\right) f\left(y_t^1\right) dy_t^1 + \delta E_{X_{t+1}^1, X_t^{-1}, P_t} c(x_{t+1}^1, \gamma, p_t, x_t^{-1})$. Since the equilibrium is assumed to be stationary, the functional form of the expected continuation payoff $E_{X_{t+1}^1, X_t^{-1}, P_t} c\left(x_{t+1}^1, z, p_t, x_t^{-1}\right)$ will be the same for any period t. Then standard FOC w.r.t. γ plus setting γ to x_t^1 method gives us a constant price path. Q.E.D.

Notes

1. Weber (1983) produces a constant price trend too but in a finite-stage, single-unit demand setting.
2. Notice that the declining price anomaly refers to the declining bidding prices in a sequential auction, not the resale prices we mention here.
3. Notice that here bidders are the investors who aim for the resale of the objects to the final consumers.

4. We can make more general assumption for P_t like $P_t = \hat{U}(\Theta_t, X_t^1, X_t^2, ...X_t^N)$. While in such situations, once $\hat{U}(.)$ is not linear in $\Theta_t, \hat{U}(\Theta_t,.)$ tends to exhibit certain trend even when Θ_t is a martingale. For example, if $\hat{U}(.)$ is concave or convex in Θ_t, $\hat{U}(\Theta_t,.)$ turns out to be a submartingale or supermartingale respectively. To avoid a price trend brought by the evolution of the fundamental value itself rather than bidders' strategic interactions, we choose to focus on $\hat{U}(.)$ being linear in Θ_t in this paper.
5. In the symmetric auctions with iid private values, it has been shown that there exists a unique equilibrium which is monotonic. However, in many variant auction setups, the existence and uniqueness of equilibrium is generally not guaranteed. In our model, the uniqueness of symmetric equilibrium is unknown and difficult to verify. Therefore, this paper chooses to focus on the most intuitive equilibrium among all the symmetric ones, i.e., the symmetric monotonic equilibrium.
6. Since we can freely specify the off-equilibrium path belief, this assumption helps to restrict the equilibrium bids to a closed interval in the simplest way.
7. Note: the continuation payoff from period t + 1 remains the same no matter if there is mimicking in period t or not because the inference of z at period t + 1 will not be affected.

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